Acta Math. Hungar.
105 (3) (2004), 249–256.

PRIME AND COMPOSITE NUMBERS AS INTEGER PARTS OF POWERS

G. ALKAUSKAS and A. DUBICKAS* (Vilnius)

Abstract. We study prime and composite numbers in the sequence of integer parts of powers of a fixed real number. We first prove a result which implies that there is a transcendental number $\xi > 1$ for which the numbers $[\xi^{n!}]$, $n = 2, 3, \dots$, are all prime. Then, following an idea of Huxley who did it for cubics, we construct Pisot numbers of arbitrary degree such that all integer parts of their powers are composite. Finally, we give an example of an explicit transcendental number ζ (obtained as the limit of a certain recurrent sequence) for which the sequence $[\zeta^n]$, $n = 1, 2, \dots$, has infinitely many elements in an arbitrary integer arithmetical progression.

1. Introduction

One of the oldest questions in number theory is to determine which integer sequences contain infinitely many prime numbers. In this paper, we are interested in sequences which are integer parts of powers of a fixed real number $[\xi^n]$, $n \in \mathbf{N}$. Usually, the sparser the sequence is the more difficult the problem becomes, but sometimes the situation is precisely opposite. The following theorem is a generalization of an old result of Mills [9] who showed that there is a $w > 1$ such that the numbers $[w^{3^n}]$, $n \in \mathbf{N}$, are all prime. (See also [10] for another prime representing function.)

THEOREM 1. *Let $A_1 = 1 < A_2 < A_3 < \cdots$ be a sequence of positive integers satisfying $A_n > 2.1053A_{n-1}$ for $n = 2, 3, \dots$. Then there exists $w > 1$ such that $[w^{A_n}]$, $n \in \mathbf{N}$, are all prime. If, in addition, $\limsup_{n\to\infty} A_n/A_{n-1} = \infty$ then w can be chosen to be transcendental.*

COROLLARY. *There is a transcendental number $w > 1$ such that the numbers $[w^{n!}]$, $n = 2, 3, \dots$, are all prime.*

---

*This research was partially supported by the Lithuanian State Science and Studies Foundation.
Key words and phrases: transcendental numbers, Pisot numbers, prime and composite numbers.
2000 Mathematics Subject Classification: 11A41, 11J81, 11R06.

0236–5294/4/$ 20.00

Baker and Harman [2] proved that the sequence $[\xi^p]$ (where p runs over the primes) contains infinitely many prime numbers for almost all $\xi > 1$. On the other hand, there are uncountably many ξ such that $[\xi^n]$ are composite for all $n \in \mathbf{N}$ [2]. Their method does not allow to give an explicit value for such ξ. Nevertheless, Huxley [2, p. 80] pointed out that, for every $n \in \mathbf{N}$, $\left[\left(1/(2^{1/3}-1)^2\right)^n\right]$ is either $6k+2$ or $6k+3$ with $k \in \mathbf{N}$, and so it is composite. In this direction, we construct algebraic numbers α of arbitrary degree d such that $[\alpha^n]$, $n \in \mathbf{N}$, are all even and so composite. (Evidently, one can take $\alpha = 4$ for $d = 1$.) Recall that $\alpha > 1$ is a Pisot number if it is an algebraic integer whose conjugates are all smaller than 1 in absolute value.

THEOREM 2. *Let $d \geqq 2$ be an integer and let α be the largest positive root of $z^d - 3Bz^{d-1} + 2Bz^{d-2} + 2 = 0$, where $B > 3(2d)^{d-2}$ is an odd integer. Then α is a Pisot number and the numbers $[\alpha^n]$, $n \in \mathbf{N}$, are all even and $\geqq 14$; so they all are composite.*

A result of Koksma [8] implies that the sequence $[\xi^n]$, $n \in \mathbf{N}$, contains infinitely many composite numbers for almost all $\xi > 1$. However, just a few explicit such ξ are known. Forman and Shapiro [6] proved that the sequences $\left[(3/2)^n\right]$ and $\left[(4/3)^n\right]$ $(n \in \mathbf{N})$ both contain infinitely many composite numbers (see also Problem E19 in [7]). The second named author [5] proved that this is also the case with the sequence $[\alpha^n]$, $n \in \mathbf{N}$, where α is a Pisot or a Salem number. (Earlier, Cass [3] proved this result for quadratic Pisot numbers.) All these explicit examples are with algebraic numbers: no explicit transcendental numbers ξ for which $[\xi^n]$ is composite for infinitely many n were known. Our next result gives an example of such a number (obtained as the limit of a certain recurrent sequence). Using the lemma (see Section 2) below one can construct many similar examples.

THEOREM 3. (i) *Let $x_1 = 2$ and $x_n = x_{n-1}^n + 2$ for $n = 2, 3, \ldots$. Then $\lim_{n\to\infty} x_n^{1/n!} = \xi = 2.4532553179\ldots$ exists, it is transcendental, and the numbers $[\xi^{n!}]$ are even for every $n \in \mathbf{N}$.*

(ii) *Let $x_1 = 2$ and $x_n = x_{n-1}^n + n - 1$ for $n = 2, 3, \ldots$. Then $\lim_{n\to\infty} x_n^{1/n!} = \nu = 2.2419914585\ldots$ exists, it is transcendental, and the sequence $[\nu^n]$, $n \in \mathbf{N}$, contains infinitely many elements divisible by an arbitrary prime number p.*

Part (ii) of the theorem proves much more than required for the problem concerning composite numbers. However the next theorem is even more general. We construct a "universal" explicit ζ such that the set $[\zeta^n]$, $n \in \mathbf{N}$, has an infinite intersection with an arbitrary integer arithmetical progression.

THEOREM 4. *Let a_n be a sequence of positive integers such that $a_1 = 1$ and $a_n \geqq n^{a_1 \cdots a_{n-1}}$ for $n = 2, 3, \ldots$. If $x_1 = 2$ and $x_n = n! x_{n-1}^{a_n} + n$ for*

$n = 2, 3, \dots,$ *then* $\lim_{n\to\infty} x_n^{1/a_1\dots a_n} = \zeta$ *exists. The number* ζ *is transcendental and the sequence* $[\zeta^n]$, $n \in \mathbf{N}$, *has infinitely many elements in an arbitrary integer arithmetical progression.*

In case if a_n in the theorem is defined by $a_n = n^{a_1\cdots a_{n-1}}$ for every $n \geqq 2$ we have $\zeta = 3.4932577001\dots$.

We conclude with the most general theorem which shows that arithmetical progressions can be replaced by arbitrary infinite sets of positive integers. Naturally, as in case of Theorem 1, this is an existence result.

THEOREM 5. *Let* $L_k = \{s_{k1} < s_{k2} < s_{k3} < \cdots\}$, $k \in \mathbf{N}$, *be a countable set of arbitrary infinite sequences of positive integers. Then there is a transcendental number* γ *such that* $[\gamma^n]$, $n \in \mathbf{N}$, *has infinitely many elements in every* L_k.

On the other hand, there exists a countable set of infinite sequences L_k, $k \in \mathbf{N}$, such that for any algebraic $\gamma > 1$ the set $[\gamma^n]$, $n \in \mathbf{N}$, has no elements in at least one L_k. Indeed, let β_i, $i = 1, 2, \dots,$ be all real algebraic numbers greater than 1. For every i, let k_i be a positive integer such that $\beta_i^{k_i}(\beta_i - 1) > 2$. Set $L_i = \{[\beta_i^{k_i}] + 1, [\beta_i^{k_i+1}] + 1, [\beta_i^{k_i+2}] + 1, \dots\}$. If $\gamma = \beta_k$ then no element of $[\gamma^n]$, $n \in \mathbf{N}$, belongs to L_k.

In the next section we give the proof of our main lemma which is used for all other proofs (except for the proof of Theorem 2 which is based on different ideas). In Section 3 we prove the theorems.

2. Transcendency of the limit of a recurrent sequence

LEMMA. *Let* $a_1 = 1$ *and let* $a_2, a_3, \dots$ *be a sequence of rational numbers greater than* 1 *such that* $A_n = a_1 \dots a_n \in \mathbf{N}$ *for every* $n \in \mathbf{N}$. *Suppose that* $x_1, x_2, x_3, \dots$ *are positive integers such that*

$$x_{n-1}^{a_n} \leqq x_n \leqq x_{n-1}^{a_n} + a_n x_{n-1}^{b_n} - 1,$$

where b_n *are some real numbers satisfying* $0 \leqq b_n \leqq a_n - 1$ *for* $n = 2, 3, \dots$. *Then*

$$\lim_{n\to\infty} x_n^{1/A_n} = \xi,$$

exists and $[\xi^{A_n}] = x_n$ *for every* $n \in \mathbf{N}$.

Moreover, if $x_{n-1}^{a_n} < x_n$ *for infinitely many* n *and* $\limsup_{n\to\infty}(a_n - b_n) = \infty$, *then* ξ *is a transcendental number.*

PROOF. Clearly, the sequence x_n^{1/A_n} is nondecreasing in n, whereas the sequence $(1+x_n)^{1/A_n}$ is decreasing, because

$$(1+x_{n-1})^{a_n} > x_{n-1}^{a_n} + a_n x_{n-1}^{a_n-1} \geqq x_{n-1}^{a_n} + a_n x_{n-1}^{b_n} \geqq 1 + x_n.$$

Since the nth element of the first sequence is smaller than the nth element of the second, every element of the first sequence is strictly smaller than an arbitrary element of the second. In particular, every element of the first sequence is bounded above by x_1+1, whereas every element of the second sequence is bounded from below by x_1. Both sequences thus tend to certain limits, say, ξ and ξ^*. The inequalities $x_n^{1/A_n} \leqq \xi \leqq \xi^* < (1+x_n)^{1/A_n}$ imply that $x_n \leqq \xi^{A_n} < 1 + x_n$ for every $n \in \mathbf{N}$, so $[\xi^{A_n}] = x_n$.

We are left to show that with the additional conditions ξ is transcendental. For the contradiction, assume that ξ is an algebraic number of degree d. Recall that the Mahler measure $M(\xi)$ of ξ is defined as $u(\xi)\prod_i \max\left\{1, \left|\xi^{(i)}\right|\right\}$, where the product runs over every conjugate $\xi^{(i)}$ of ξ and where $u(\xi) \in \mathbf{N}$ is the leading coefficient of the minimal polynomial of ξ. Consider the positive algebraic number $\beta_n = x_{n-1}^{1/A_{n-1}}$. Let D be the degree of β_n over $\mathbf{Q}$ and let $\beta_n^{(j)}$, $1 \leqq j \leqq D$, be conjugate to β_n. They are all of equal modulus. Clearly, $\beta_n < \xi$. Hence no conjugate of β_n is equal to ξ.

Consider the resultant of ξ and β_n (which, by the above, is nonzero)

$$R(\xi, \beta_n) = u(\xi)^D u(\beta_n)^d \prod_{i,j} \left|\xi^{(i)} - \beta_n^{(j)}\right|,$$

where the product which contains Dd terms is taken over every pair of conjugates and where n is so large that $\beta_n = x_{n-1}^{1/A_{n-1}} > \xi - \delta > 1 + \delta$ for some absolute positive constant δ. The polynomial $X^{A_{n-1}} - x_{n-1}$ vanishes at β_n. Hence it is divisible by the minimal polynomial of β_n and $D \leqq A_{n-1}$. By the multiplicative property of the Mahler measure for polynomials, we have $M(\beta_n) \leqq x_{n-1}$. Since $\xi > \beta_n$, by estimating every difference $\left|\xi^{(i)} - \beta_n^{(j)}\right|$ (except for $|\xi - \beta_n|$) by $2\max\left\{1, \left|\xi^{(i)}\right|\right\}\max\left\{1, \left|\beta_n^{(j)}\right|\right\}$, we deduce that

$$1 \leqq \left|R(\xi, \beta_n)\right| \leqq |\xi - \beta_n| 2^{dD-1} M(\xi)^D M(\beta_n)^d < |\xi - \beta_n| \left(2^d M(\xi)\right)^{A_{n-1}} x_{n-1}^d.$$

Using the inequality $(1+x_n)x_{n-1}^{-a_n} \leqq 1 + a_n x_{n-1}^{b_n - a_n}$, we obtain that

$$0 < |\xi - \beta_n| = \xi - \beta_n < (1+x_n)^{1/A_n} - x_{n-1}^{1/A_{n-1}}$$

$$= \beta_n\Big(\big((1+x_n)x_{n-1}^{-a_n}\big)^{1/A_n} - 1\Big) \leqq \beta_n\Big(\big(1+a_n x_{n-1}^{b_n-a_n}\big)^{1/A_n} - 1\Big)$$

$$< \beta_n(a_n/A_n)x_{n-1}^{b_n-a_n} \leqq \beta_n x_{n-1}^{b_n-a_n}.$$

Set $c = 2^d M(\xi)$. It follows that

$$1 < \beta_n x_{n-1}^{b_n-a_n+d} c^{A_{n-1}} = \beta_n\big(\beta_n^{b_n-a_n+d} c\big)^{A_{n-1}} \leqq \big(\beta_n^{b_n-a_n+d+1} c\big)^{A_{n-1}}.$$

But $\beta_n > 1+\delta$ and $\limsup_{n\to\infty}(a_n - b_n) = \infty$, so the right–hand side will be smaller than 1 for certain large integer n, a contradiction.

3. Proofs

Proof of Theorem 1. Let x_1 be a sufficiently large prime number. It was shown in [1] that there is always a prime in the interval $(x, x + x^{0.525})$ for every sufficiently large x. Set $a_n = A_n/A_{n-1}$ and $b_n = 0.525a_n$. Then $a_n - 1 > 0.525a_n = b_n$. It follows that there is a sequence of prime numbers $x_1 < x_2 < x_3 < \cdots$ such that

$$x_{n-1}^{a_n} < x_n < x_{n-1}^{a_n} + a_n x_{n-1}^{b_n} - 1.$$

By applying the lemma we get the first part. The second part also follows from the lemma, because $\limsup_{n\to\infty}(a_n - b_n) = \limsup_{n\to\infty} 0.475A_n/A_{n-1} = \infty$. □

Proof of Corollary. Set $A_n = (n+1)!/2$ and replace w by w^2 in Theorem 1. □

Proof of Theorem 2. Set $P(z) = z^d - 3Bz^{d-1} + 2Bz^{d-2} + 2$. On the circle $|z| = 1/2d$, $|2Bz^{d-2}| > |z^d - 3Bz^{d-1} + 2|$, so by Rouché's theorem $P(z)$ has $d-2$ roots in the circle $|z| \leqq 1/2d$, say $\alpha_3, \ldots, \alpha_d$. Note that $P(2/3) > 0$ and

$$P\left(1 - \frac{1}{4d}\right) \leqq \left(1 - \frac{1}{4d}\right)^d - \frac{21B}{8}\left(1 - \frac{1}{4d}\right)^{d-2}$$

$$+ 2B\left(1 - \frac{1}{4d}\right)^{d-2} + 2 < 3 - \frac{5B}{8}\left(1 - \frac{1}{4d}\right)^{d-2}$$

which is negative (check this for $d = 2$ and for $d \geqq 3$). Hence the polynomial $P(z)$ has a root in the interval $(2/3, 1 - 1/4d)$, say, α_2. Similarly, using

$P(3B-1) = 2-(B-1)(3B-1)^{d-2} < 0$ and $P(3B) > 0$, we see that it has a root in $(3B-1, 3B)$ which we denote by $\alpha = \alpha_1$. Since α is a unique root of $P(z)$ outside the unit circle, it is a Pisot number. (Moreover, it is a strong Pisot number in the sense of [4].) It follows that $[\alpha] = 3B-1$ is even and $\geqq 14$.

Set $S_k = \alpha^k + \alpha_2^k + \alpha_3^k + \dots + \alpha_d^k$. We claim that $[\alpha^k] = S_k - 1$ for every $k \geqq 2$. Indeed, $|\alpha_3^k + \dots + \alpha_d^k| \leqq (d-2)(2d)^{-k}$ and $(2/3)^k < \alpha_2^k < (1-1/4d)^k$. It is easy to check that $(d-2)(2d)^{-k} < (2/3)^k$ and $(d-2)(2d)^{-k} + (1-1/4d)^k < 1$ for $k \geqq 2$, so $S_k - \alpha^k$ is positive and < 1 which completes the proof of the claim, because $S_k \in \mathbf{N}$.

Finally, we will show that S_k, $k = 2, 3, \dots$, are all odd. Write $P(z) = z^d - \sigma_1 z^{d-1} + \sigma_2 z^{d-2} - \dots + (-1)^d \sigma_d$. By Waring's formula, we can express S_k as

$$S_k = \sum \ell_{k_1,\dots,k_d} \sigma_1^{k_1} \dots \sigma_d^{k_d},$$

where $k_1 + 2k_2 + \dots + dk_d = k$, $\ell_{k,0,\dots,0} = 1$ and $\ell_{k_1,\dots,k_d}$ are integers. Every term in the above expression except for σ_1^k contains at least one σ_j with $j \geqq 2$. But σ_j, $2 \leqq j \leqq d$, are all even, so $S_k - \sigma_1^k = S_k - (3B)^k$ is even. Hence S_k is odd for every $k \geqq 2$. $\square$

REMARK 1. By the same method we can show, for instance, that the numbers $\left[\left(\frac{5+\sqrt{17}}{2}\right)^n\right]$, $n \in \mathbf{N}$, are all even. However it is not known whether there are finitely or infinitely many primes in the Fibonacci sequence $f_n = \left[\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{1}{2}\right]$, $n \in \mathbf{N}$.

PROOF OF THEOREM 3. Part (i) follows immediately from the lemma setting $a_n = n$, $b_n = 1$. Then $A_n = n!$ and $[\xi^{n!}] = x_n$ are all even.

For (ii), setting $a_n = n$, $b_n = 0$, $A_n = n!$ and applying the lemma, we see that $\lim_{n\to\infty} x_n^{1/n!} = \nu$ is transcendental. Since $[\nu^{n!}] = x_n$, it suffices to show that the sequence $x_1, x_2, x_3, \dots$ contains infinitely many elements divisible by an arbitrary prime number p. Set $n = p(p-1)k$, where $k \in \mathbf{N}$. We claim that either x_{n-1} or x_n is divisible by p. Indeed, if x_{n-1} is not divisible by p, then by the Little Fermat Theorem $x_{n-1}^n = x_{n-1}^{p(p-1)k}$ is equal to 1 modulo p. Hence $x_n = (x_{n-1}^n - 1) + p(p-1)k$ is divisible by p. $\square$

PROOF OF THEOREM 4. We cannot apply the lemma directly because of the factor $n!$, but follow its proof. Set $A_n = a_1 \dots a_n$. Firstly, the sequence x_n^{1/A_n} is increasing, because $x_n > x_{n-1}^{a_n}$. Secondly, the sequence $(1+x_n)^{1/A_n}$ is decreasing for $n \geqq 6$. Indeed, we have that

$$1 + x_n = 1 + n!x_{n-1}^{a_n} + n \leqq 2n!x_{n-1}^{a_n} < n^n x_{n-1}^{a_n}.$$

We need to show that this is less than $(1 + x_{n-1})^{a_n}$ which would follow from the inequality $a_n > 2x_{n-1}n \log n$. In order to prove this inequality note that $x_n \leqq 2n!x_{n-1}^{a_n} < (2x_{n-1})^{a_n}$. Hence $\log x_n < a_n(\log 2 + \log x_{n-1})$, giving

$$\log x_n < A_n(1 + 1/A_1 + 1/A_2 + \cdots + 1/A_{n-1}) \log 2 < A_n \log 6.$$

Consequently, $x_n < 6^{A_n}$. We are left to show that $a_n \geqq n^{A_{n-1}} > 6^{A_{n-1}} 2n \log n$ for $n \geqq 7$ which follows from $A_{n-1} > 13n \log n$ (for $n \geqq 7$). This proves that $\lim_{n\to\infty} x_n^{1/A_n} = \zeta$ exists.

By the above $\zeta > x_{n-1}^{A_{n-1}}$. In order to show that ζ is transcendental it suffices to prove that, given any absolute positive constants c and d,

$$(\zeta - x_{n-1}^{1/A_{n-1}}) c^{A_{n-1}} x_{n-1}^d < 1$$

for n sufficiently large. Note that $\zeta - x_{n-1}^{1/A_{n-1}} < 6\big((2n!)^{1/A_n} - 1\big)$ which is less than $n^2/A_n < 1/a_n$ for n sufficiently large. But $c^{A_{n-1}} x_{n-1}^d < (c6^d)^{A_{n-1}}$ $< n^{A_{n-1}} \leqq a_n$ for large n, giving the required inequality. This shows that ζ is transcendental and $[\zeta^{A_n}] = x_n$ for every $n \geqq 6$.

Let $uy + v$, $y = 1, 2, \dots$, be an integer arithmetical progression with integers $u > 0$ and $v \geqq 0$. Setting $n = uy + v$ with y sufficiently large, we see that $x_n = n!x_{n-1}^{a_n} + n$ is of the form $uy' + v$, where $y' \in \mathbf{N}$, so it belongs to the same arithmetical progression. This completes the proof. $\square$

REMARK 2. The inequality $a_n \geqq n^{a_1 \cdots a_{n-1}}$ for the sequence a_n, $n = 2$, $3, \dots$ (in Theorem 4) can be replaced by $\lim_{n\to\infty} (\log a_n)/a_1 \dots a_{n-1} = \infty$.

PROOF OF THEOREM 5. Take $x_1 \in L_1$, $x_2 \in L_1$, $x_3 \in L_2$, $x_4 \in L_1$, $x_5 \in L_2$, $x_6 \in L_3, \dots$, where $x_{n-1}^{a_n} < x_n < x_{n-1}^{a_n+2}$. Here $a_1 = 1$, a_n, $n = 2$, $3, \dots$, is a sequence of positive integers such that $a_n \geqq n^{a_1 \cdots a_{n-1}}$ and $x_{n-1}^{a_n+2}$ $< (1 + x_{n-1})^{a_n} - 1$. The possibility to choose x_n in any L_k exists, because the intervals $(x_{n-1}^s, x_{n-1}^{s+2})$, $s = N, N+1, N+2, \dots$, overlap and their union contains all sufficiently large integers.

Setting $A_n = a_1 \dots a_n$, we see that the sequence x_n^{1/A_n}, $n = 1, 2, \dots$, is increasing and the sequence $(1 + x_n)^{1/A_n}$, $n = 1, 2, \dots$, is decreasing. Set $\gamma = \lim_{n\to\infty} x_n^{1/A_n}$. Clearly, $x_1 < \gamma < x_1 + 1$. Since

$$\gamma < (1 + x_n)^{1/A_n} < \big(1 + x_{n-1}^{a_n+2}\big)^{1/A_n} < x_{n-1}^{(a_n+3)/A_n} = \big(x_{n-1}^{1/A_{n-1}}\big)^{1+3/a_n}$$

and $x_{n-1}^{1/A_{n-1}} < x_1 + 1$, we have that

$$\gamma - x_{n-1}^{1/A_{n-1}} < (x_1+1)\bigl((x_1+1)^{3/a_n} - 1\bigr).$$

For n sufficiently large, this is less than c'/a_n, where c' is an absolute constant. As in the proof of Theorem 4, $c^{A_{n-1}} x_{n-1}^d < g^{A_{n-1}}$ with an absolute constant g. Hence $0 < \bigl(\gamma - x_{n-1}^{1/A_{n-1}}\bigr) c^{A_{n-1}} x_{n-1}^d < 1$ for n sufficiently large, so γ cannot be algebraic. Thus it is transcendental which completes the proof. □

References

[1] R. C. Baker, G. Harman and J. Pintz, The difference between consecutive primes. II, *Proc. London Math. Soc.,* **83** (2001), 532–562.
[2] R. C. Baker and G. Harman, Primes of the form $[c^p]$, *Math. Zeitschr.,* **221** (1996), 73–81.
[3] D. Cass, Integer parts of powers of quadratic units, *Proc. Amer. Math. Soc.,* **101** (1987), 610–612.
[4] A. Dubickas, A note on powers of Pisot numbers, *Publ. Math. Debrecen,* **56** (2000), 141–144.
[5] A. Dubickas, Integer parts of powers of Pisot and Salem numbers, *Archiv der Math.,* **79** (2002), 252–257.
[6] W. Forman and H. N. Shapiro, An arithmetic property of certain rational powers, *Comm. Pure Appl. Math.,* **20** (1967), 561–573.
[7] R. K. Guy, *Unsolved Problems in Number Theory,* Springer–Verlag (New York, 1994).
[8] J. F. Koksma, Ein mengen-theoretischer Satz über Gleichverteilung modulo eins, *Compositio Math.,* **2** (1935), 250–258.
[9] H. W. Mills, A prime representing function, *Bull. Amer. Math. Soc.,* **53** (1947), 604.
[10] E. M. Wright, A prime representing function, *Amer. Math. Monthly,* **58** (1951), 616–618.

(Received May 13, 2003)

DEPARTMENT OF MATHEMATICS AND INFORMATICS
VILNIUS UNIVERSITY
NAUGARDUKO 24
LT-03225 VILNIUS
LITHUANIA
E-MAILS: GIEDRIUS.ALKAUSKAS@MAF.VU.LT
ARTURAS.DUBICKAS@MAF.VU.LT