

QUESTION FROM COMPLEX DYNAMICS

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1. FORMULATION

Let \mathcal{A} and \mathcal{B} be two analytic maps of the Riemann sphere, and let α , β and γ be (finite or infinite) complex numbers such that

$$\mathcal{A}(\alpha) = \alpha, \quad \mathcal{B}(\beta) = \beta, \quad \mathcal{A}(\beta) = \mathcal{B}(\alpha) = \gamma.$$

Describe the necessary and sufficient conditions in order for the closure of the set of points $\gamma, \mathcal{A}(\gamma), \mathcal{B}(\gamma), \mathcal{A} \circ \mathcal{A}(\gamma), \mathcal{A} \circ \mathcal{B}(\gamma), \mathcal{B} \circ \mathcal{A}(\gamma), \mathcal{B} \circ \mathcal{B}(\gamma) \dots$ to be a continuous curve. Examples:

(i) $\mathcal{A}(z) = pz^2$, $\mathcal{B}(z) = cz + \frac{(1-c)^2}{p}$. Here $\gamma = \frac{(1-c)^2}{p}$. Figure 1 shows the locus points for $p = 1 + 0.9i$, $c = 0.6 + 0.2i$.

(ii) $\mathcal{A}(z) = pz^2$, $\mathcal{B}(z) = (p-1)(z-1)^2 + 1$. Here $\gamma = p$. Figure 2 shows the locus points for $p = 0.5 + 0.01i$.

While some choices of the parameters do work, other produce chaotic sets of points.

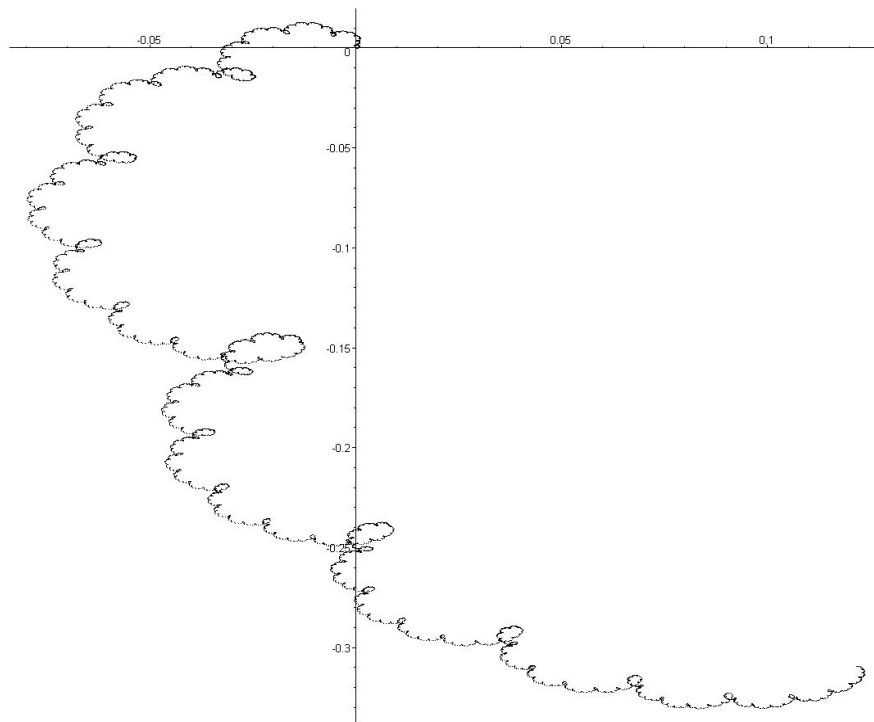


FIGURE 1

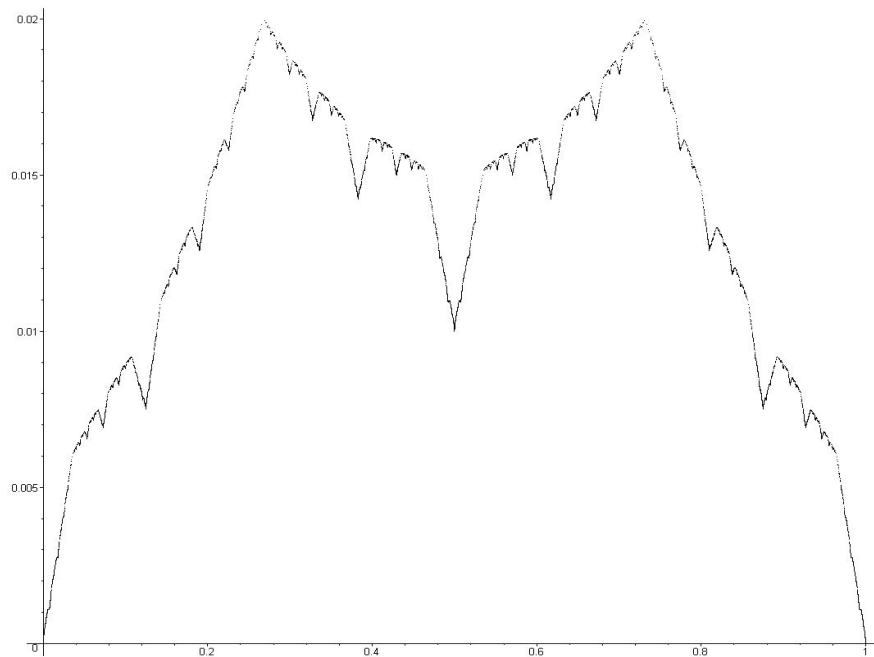


FIGURE 2