My previous and latest research interests

My doctoral research has been centered on heights of single-variable polynomials and their applications. The *height* of a polynomial

$$P(x) = a_n x^n + \cdots + a_1 x + a_0 = a_n(x - \alpha_1)\dots(x - \alpha_n)$$

with real or complex coefficients measures the complexity of the polynomial P. There are several different types of heights. The most simple heights take into account the size of coefficients of a polynomial P, such as the *naive* height $H(P)$. Another height, which is called the *Mahler measure* $M(P)$ uses the size of the roots of polynomial P as a reference. There are heights which are based on the values of the polynomial P in some compact subset of the complex plane, for instance, the integral L_s norm which is denoted by $||P||_s$. Heights of polynomials have a large number of applications in many different areas. Just to mention a few: bounds for linear forms in logarithms (diophantine equations), arithmetic properties of algebraic numbers (algebraic number theory), binary sequences with minimal autocorrelations (signal processing, coding theory). The given list of examples is far from being complete.

During my doctoral studies [11], I was interested how the height of polynomial P affects the analytic properties of polynomial (such as the extremal values), the divisibility and reducibility properties of integer polynomials in $\mathbb{Z}[x]$. I investigated the arithmetic and algebraic properties of roots of polynomial P that depend on the height of P. I did research on the zero sets of Newman and Littlewood polynomials, the number of their roots inside and outside the unit circle. I have studied lower bounds on Mahler measures and L_s norms of derivatives of reciprocal polynomials and polynomials related to Barker sequences.

In my post-doc years, my attention shifted to number–theoretical problems where my skills in polynomials can be applied in a combination with techniques originating in Galois theory or Diophantine analysis. More specifically, I did research on the distribution of fractional parts of Pisot numbers with minimal polynomials of small length [9]; I investigated the solutions to simple 3 and 4-term linear equations in conjugates of a Pisot number [5] or conjugates of a arbitrary algebraic numbers of small degree [10]. My latest research ventured into the combinatorial properties of words that represent the interlaced roots of pairs of polynomials that appear in Pisot–Salem correspondence problem [12].

Publications and scientific collaboration

The results of my doctoral and a post-doctoral have been published in **16 papers** in general or specialized periodical peer reviewed mathematical journals, such as: *Proceedings of AMS*, *Bulletin of LMS*, *Journal of Australian Math. Soc.*, *Bulletin of Brazilian Math. Soc.*, *Mathematics of Computation*, *Acta Arithmetica*, *Journal of Number Theory*, etc. **Two more** manuscripts are submitted for the review and publication, **one** project is still in progress. I am well accustomed to the work in a team of mathematicians. I had a number of successful collaborations with professors Shigeki Akyama, Peter Borwein, Stephen Choi, Artūras Dubickas (my doctoral advisor), Kevin Hare, my colleagues Paulius Drungilas, Charles Samuels and Himadri Ganguli. In all the cases, I have been able to make contributions to the results of a joint work. My selected results are:

- Sharp bounds on Mahler measures and L_s norms of reciprocal polynomials with large central coefficient related to Barker problem [2], [3].
- Tight bound for the size of an intersection of real geometric and arithmetic progressions [6].
- An improved inequality for the Mahler measure of the derivative of a self inversive polynomial [7], [11].
- Results on the zero sets of Newman and Littlewood polynomials [8].
- Solution of 3 and 4-term linear equations in conjugates of a Pisot number [5].

Research proposal in number systems and fractals

Let $\alpha \in \mathbb{C}$ be an algebraic integer and let $\mathcal{B} \neq \varnothing$ *be a finite* subset of integers $\mathbb{Z}$. The set

$$\mathcal{B}[\alpha] = \{b_0 + b_1\alpha + \cdots + b_n\alpha^n \mid b_j \in B, 0 \leq j \leq n\}$$

of numbers that can be represented as polynomials in α (*the base*) with integer coefficients (or *digits*) from $\mathcal{B}$ is called *the number system in* $\mathbb{Z}[\alpha]$. This is a generalization of the usual number systems in integral bases. When all digits b_j satisfy $0 \leq b_j \leq \mathrm{N}(\alpha) - 1$, the number system is called *canonical* (CNS). The central questions in the theory of number systems are:

1) What elements of the ring $\mathbb{Z}[\alpha]$ can be expressed in the particular number system?
2) Are such expressions unique? If not, how one obtains other possible expressions? Can they be obtained by using a finite automata?
3) When $\mathbb{Z}[\alpha] = \mathcal{B}[\alpha]$?
4) What kind of numbers α produce a CNS?
5) When the expansions in the number systems $\mathcal{B}[[\alpha^{-1}]]$ (that generalize the a fractional part expansion of real numbers) are unique and/or periodic, especially when doing a greedy or lazy expansion in Pisot or Salem bases α?

It turns out, these questions on the number systems have deep connection to the tilings of $\mathbb{R}^d$ obtained from the $\mathcal{B}$-translates of the fractals produced by the companion matrix of the minimal polynomial of α.

By using a language of fractal tilings and Fourier analysis, Lagarias and Wang [15, 16, 17] established that every *expanding* algebraic integer α produce a number system such that $\mathbb{Z}[\alpha] = \mathcal{B}[\alpha]$. While their proof establishes the existence, no efficient deterministic algorithm to produce the expansions is known except the one that reduce the problem to an exhaustive search in a finite set. An efficient algorithm would be very important in proving the tiling properties of other fractals in aperiodic cases where the Fourier analysis breaks down.

Another interesting problem in the number systems is whether $\alpha^n \in \mathcal{B}[\alpha]$ for all sufficiently large integers n. This is equivalent to the fact that the minimal polynomial $P(x)$ of α divides some monic polynomial from $\mathbb{Z}[x]$ with non-leading coefficients in $-\mathcal{B}$. For expanding algebraic integers α, the expected answer is positive. In the language of fractals, this would imply the connectivity conjecture made by Kirat, Lau and Wang [13, 14].

My recearch experience on the topic includes a collaboration on two papers [1, 8] were we used computer algorithms to find multiples of a given polynomial $P \in \mathbb{Z}[x]$ with coefficients in the prescribed set $\mathcal{B}$, or to find the expansions of a given element $\alpha \in \mathbb{Z}[\alpha]$ in the number system $\mathcal{B}[\alpha]$. One more computational project that generalizes the algorithm and extends the result of [8] is still ongoing. So far, my point of view to this number system problem was that of a pure arithmetics in $\mathbb{Z}[\alpha]$ (or $\mathbb{Z}[x]$); I did not attempt to exploit the underlying fractal properties since I have no background in fractal geometry and tilings.

The main goal of my post–doc research would be to develop more efficient algorithm for finding expansions in number systems or multiples of a polynomials with restricted coefficients for various classes of algebraic integers (expanding, Pisot, Salem cases) by exploiting the corresponding geometric properties of fractals. The basic steps in achieving this goal would be:

- To acquire the necessary background in fractal geometry and dynamical systems and get accustomed with the standard tools used in the theory (Hausdorff measure and dimension, substitution systems, finite automata).
- To understand the relationship between the arithmetics in number systems $\mathcal{B}[\alpha]$ and the underlying geometric properties of fractals produced by linear transformations with integer matrices via the machinery developed by Lagarias–Wang [15, 16, 17] and Kirat–Lau–Rao [13, 14] and Gröeschenig–Haas [4] contact matrix theory.
- Try to find and prove geometrical properties of fractals and their arithmetical analogues.
- Write computer code that exploits fractal geometry in the arithmetical setting.

I assume that my post-doctoral research would consist of mathematical research and programming under the guidance of a scientist with a considerable research experience in the areas related to number systems, fractals and dynamical systems. In addition to this, I would like to express my sincere interest to work on other topics related to my research or topics which are currently outside the scope of my mathematical interests. I am confident that my experience working with colleagues as a part of research team is one of my strong sides. I am looking for the opportunities to do joint research work and mathematical collaboration in the accepting institution.

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