On a class of polynomials related to Barker sequences

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2011

Unimodular sequences and autocorrelations

- A **unimodular sequence**:

$$a_0 \quad a_1 \quad a_2 \quad \dots \quad a_n$$

$$a_j \in \mathbb{C}, \qquad |a_j| = 1.$$

- For such a sequence, the **autocorrelation coefficients** c_k, $k = 0, 1, \dots, n$ are defined as a *dot product* of the sequence and a shift of the same sequence:

$$\begin{array}{cccccccccc} a_0 & a_1 & \dots & a_k & a_{k+1} & \dots & a_n & & & \\ & & & a_0 & a_1 & \dots & a_{n-k} & \dots & a_n \end{array}$$

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Examples

- In particular, the coefficient c_0 is called a **central autocorrelation coefficient**:

$$c_0 = |a_0|^2 + |a_1|^2 + \cdots + |a_n|^2 = n + 1 .$$

- A simple example: a sequence of length four:

$$1 \quad -1 \quad 1 \quad -1$$

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  1. All numbers a_j in this sequence are equal to -1 or 1;
  2. The sequence has minimal possible autocorrelations: $c_k \in \{-1, 0, 1\}$.
- **Applications:** Barker sequences are of considerable importance in the signal processing theory. In particular, Barker sequences are **optimal** sequences for the phase-modulated pulse compression in radar design.

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The research on Barker sequences

- All known normalized Barker sequences (since 1953) are summarized in the following table:

Length	Sequence
2	$++$
3	$++-$
4	$++-+$ and $+++-$
5	$+++-+$
7	$+++--+-$
11	$+++---+--+-$
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- A theorem by **Turyn and Storer** (1961): there are no Barker sequences of odd length > 13.

- Conjecture 1

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- Restrictions on the patterns of coefficients and possible lengths of Barker sequences were obtained by:

  Turyn (1965),
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- Computations show that no Barker sequences small even lenght exist. Current computer record belongs to Mossinghoff (2009): lengths up to $2 \cdot 10^{30}$ (with a possible exception 189260468001034441522766781604).
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Mahler measures

- The most usefull height for our purposes is a **Mahler measure**: for a polynomial

$$p(z) = a_n(z-\alpha_1)(z-\alpha_2)\dots(z-\alpha_n) \in \mathbb{C}[z]$$

$$M(p) := |a_n| \prod_{j=1}^{n} \max\{1, |\alpha_j|\}.$$

Mahler measure may be computed using Jensen's formula:

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A conjecture

To prove the non-existence of long Barker sequences of even length, we consider a two part conjecture:

Conjecture 2

1. *If $p(z) \in \mathbb{C}[z]$ is a Barker polynomial of odd degree n, then $M(p)$ is* ***extremely close*** *to its L_2 norm:*

$$\lim_{n\to\infty} \left(M(p) - \sqrt{n+1} \right) = 0.$$

2. *If a polynomial $p(z)$ has all coefficients equal to -1 or 1, then Mahler measure $M(p)$ is* ***bounded away*** *from its L_2 norm:*

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New results

Suppose that $p(z)$ is a Barker polynomial of odd degree n. Then the product $p(z)p(1/z)$ takes the form

$$P(z) = (n+1) + \sum_{\substack{k=1 \\ k\text{ – odd}}}^{n} c_k \left(z^k + \frac{1}{z^k} \right),$$

where $c_k \in \{-1, 1\}$.

Definition. Let $\mathcal{LP}_n$ be the class of polynomials of the above form. In this class, the polynomials with all coefficients $c_k = 1$ are of special interest and are denoted by $R_n(z)$:

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Theorem 1

For a polynomial $R_n \in \mathcal{LP}_n$,

$$M(R_n) > n - \frac{2}{\pi}\log n + O(1).$$

Theorem 2

The polynomials $R_n(z)$ and $R_n(-z)$ have minimal Mahler measures in $\mathcal{LP}_n$, namely, for any $P \in \mathcal{LP}_n$

$$M(P) \geqslant M(R_n).$$

Theorem 3

For $s < 1$, the polynomials $R_n(\pm z)$ have minimal L_s norms in the class $\mathcal{LP}_n$. In the other hand, R_n have maximal L_s norms in $\mathcal{LP}_n$ for $s \in [2j-1, 2j]$, $j \in \mathbb{N}$, and also for all s which are sufficiently large: $s > s_0(n)$.

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Consequences

Corollary 4

If $p(z)$ is Barker polynomial of odd degree n, then

$$M(p) > \sqrt{n - 2/\pi \log n + O(1)}.$$

This proves the first part of Conjecture 2.

However, we still need a considerable progress on the second part of the Conjecture 2.