Corrigendum

Corrigendum to “Stability in $\mathbb{D}$ of martingales and backward equations under discretization of filtration”

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The proof of Lemma 3 and, consequently, that of Theorem 2 in our paper (Coquet et al., 1998) is not correct: there is no reason for the equality

$$
\mathcal{L}\left(\mathcal{E}(\mathcal{U}_n^m + \tilde{X}_n^m | \mathcal{F}_n^m) | \mathcal{P}\right) = \mathcal{L}\left(\mathcal{E}\left(\int_0^T g_x^n(V_n^m) \, dA_x^m + X_n^m | \mathcal{F}_n^m\right) | \mathcal{P}\right),
$$

at the end of the proof.

Using another method, however, we can get a better result than that stated in Theorem 2.

In what follows, the notation of Coquet et al. (1998) is employed. The hypotheses given in Section 3 of Coquet et al. (1998) remain unchanged, except for (Hₚ) where we assume that the Markov process $Y$ is càdlàg and not necessarily continuous.

**Theorem 1.** Suppose hypotheses (HX), (HA), (Hg), (Hₚ), and (Hco) are satisfied. Then the sequence $(V_n^m)$ of the solutions of equations $(\ast)$ converge to the solution $V$ of $(\ast)$ in probability for the Skorokhod topology.

**Proof.** Our method for proving this result is considering, for each $n$, the iterates $U_{n,k}$ given by Picard approximation converging, as $k \to \infty$, to the solution $V_n^m$ of $(\ast)$ and proving that $U_{n,k}$ converges in probability, as $n \to \infty$, to $U^k$, the $k$-iterated process of the Picard approximation of $V$, the solution of $(\ast)$.

To be precise, we put:

for equation $(\ast)$,

$$
U_{n,0}^m = 0,
$$

$$
U_{n,1}^m = \mathcal{E}\left(\int_t^T g_x^n(U_{n,0}^m) \, dA_x^m + X_n^m | \mathcal{F}_n^m\right)
$$

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and, by induction,
\[ U^n_{i,k} = E \left( \int_{i}^{T} g^n_s(U^n_{i,k-1}) \, dA^n_s + X^n|\mathcal{F}_i^n \right) \]
for equation (\(*\)),
\[ U^n_0 = 0, \]
\[ U^n_1 = E \left( \int_{i}^{T} g^n_s(U^n_0) \, dA_s + X_j|\mathcal{F}_i^n \right) \]
and, by induction,
\[ U^n_k = E \left( \int_{i}^{T} g^n_s(U^n_{k-1}) \, dA_s + X_j|\mathcal{F}_i^n \right). \]

**Step 1:** In Theorems 2–4 of Antonelli (1993), Antonelli proved the inequality
\[ \| U^{n,k+1} - U^{n,k} \|_{L^1(\mu_n)} \leq \frac{(c_n \beta_n)^{k+1}}{(k+1)!} \| U^{n,1} \|_{L^1(\mu_n)}, \]
where \( \mu_n(dt, d\omega) = dA^n_t(\omega)P(d\omega). \)

We deduce:
\[ \| V^n - U^{n,k} \|_{L^1(\mu_n)} \leq \sum_{p=k+1}^{\infty} \frac{(c_n \beta_n)^p}{p!} M_n, \]
where
\[ M_n = \beta_n E \left( \int_{0}^{T} |g^n_s(0)|dA^n_s + |X^n| \right). \]

Using Doob’s maximal inequality, we easily get that
\[ \forall \varepsilon > 0, \quad P \left[ \sup_{t \leq T} |V^n_t - U^{n,k+1}_t| \geq \varepsilon \right] \leq P \left[ \sup_{t \leq T} c_n E \left( \int_{0}^{T} |V^n_s - U^{n,k}_s| \, dA_s + |X^n| \right) > \varepsilon \right] \leq \frac{1}{\epsilon} c_n \| V^n - U^{n,k} \|_{L^1(\mu_n)}, \]
whence
\[ \forall \varepsilon > 0, \quad P \left[ \sup_{t \leq T} |V^n_t - U^{n,k+1}_t| \geq \varepsilon \right] \leq \frac{1}{\varepsilon} \sum_{p=k+1}^{\infty} \frac{(c_n \beta_n)^p}{p!} c_n M_n. \]

Finally, the assumptions of Theorem 1 give the uniform convergence (in \( n \)):
\[ \forall \varepsilon > 0, \quad \sup_n P \left[ \sup_{t \leq T} |V^n_t - U^{n,k}_t| \geq \varepsilon \right] \to 0, \quad k \to \infty. \] (1)
Moreover, from hypothesis (Hg), we get by induction that, for every $k$,

$$\sup_{n} E \left[ \sup_{t \in T} \left| U^{n,k}_{t} \right|^{1+\delta} \right] \leq \infty,$$

(2)

and

$$\sup_{n} E \left[ \left( \int_{0}^{T} g^{n}_{s}(U^{n,k}_{s}) \, dA^{n}_{s} \right)^{1+\delta} + \left| X^{n} \right|^{1+\delta} \right] \leq \infty.$$

(3)

**Step 2:** All convergences below are for the Skorokhod topology. From (HA) and (Hg) we get the convergence

$$\left( A^{n}, \int_{0}^{T} g^{n}_{s}(0) \, dA^{n}_{s} \right) \overset{P}{\longrightarrow} \left( A, \int_{0}^{T} g_{s}(0) \, dA_{s} \right).$$

Then, with (H.\cF) (using Theorem 1 of Antonelli (1993)), (HX), (Hco), and above inequalities (2) and (3), we get

$$\left( A^{n}, \int_{0}^{T} g^{n}_{s}(0) \, dA^{n}_{s}, E \left[ \int_{0}^{T} g^{n}_{s}(0) \, dA^{n}_{s} + X^{n} | \cF^{n} \right] \right) \overset{P}{\longrightarrow} \left( A, \int_{0}^{T} g_{s}(0) \, dA_{s}, E \left[ \int_{0}^{T} g_{s}(0) \, dA_{s} + X | \cF \right] \right).$$

Hence

$$\left( A^{n}, U^{n,1} \right) \overset{P}{\longrightarrow} (A, U^{1}).$$

We can iterate the procedure: from

$$\left( A^{n}, U^{n,k} \right) \overset{P}{\longrightarrow} (A, U^{k}),$$

we deduce, using the continuity of $g$ and the convergence of Stieltjes integrals (see for example, Jakubowski et al., 1989):

$$\left( A^{n}, \int_{0}^{T} g^{n}_{s}(U^{n,k}_{s}) \, dA^{n}_{s} \right) \overset{P}{\longrightarrow} \left( A, \int_{0}^{T} g_{s}(U^{k}_{s}) \, dA_{s} \right).$$

Using again inequalities (2), (3), and the hypotheses (HA), (H.\cF), (Hco), and (Hg) we get

$$\left( A^{n}, U^{n,k+1} \right) \overset{P}{\longrightarrow} (A, U^{k+1}).$$

Finally, for every $k$, we have the convergence for the Skorokhod topology of processes:

$$U^{n,k} \overset{P}{\longrightarrow} U^{k}.$$  

Inequality (1) of Step 1 then gives the desired result

$$V^{n} \overset{P}{\longrightarrow} V.$$  

**Remark.** Theorem 1 answers the question stated in Remark 3 following the proof of Theorem 2 in Coquet et al. (1998); however, Theorem 1 is not completely comparable.
to Proposition 3 of Coquet et al. (1998): in the latter, (Hco) is not needed, but (HA) is replaced by the stronger hypothesis of convergence in variation of $A^n$ to a deterministic $A$.

References